Simulating quantum geometries

with correlated photon pairs

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The Quantum Information Structure
of Spacetime, January 2020, Hong Kong

The quantum information structure of spacetime

in quantum gravity

Background-independent quantum gravity theories:

"Quantum states of spacetime"

- no predefined background metric → notion of distance necessarily induced by structure used to define quantum states of geometry (e.g. spin networks/foams in LQG).
- concept of localised positions becomes meaningless; superseded by relations between constituents of the geometry-defining structure.

"relational ~"

Geometric quantities (e.g. distances) must be inferred from informational concepts (e.g. correlations).

Strong link between geometry of spacetime in quantum gravity and quantum information.

The quantum information structure of spacetime

in quantum gravity

$\sim 10^{-35}$ m

Quantum gravity: at the Planck scale, structure of geometry is discrete
(i.e. area and volume operators have discrete spectra).

Problem: notoriously difficult to confirm experimentally.

+ "quantum fuzziness"

Reminder: *sharp localisation requires large momentum ($\propto$ energy);*
largest centre-of-mass energy at LHC is $E = 14$ TeV $\simeq 10^{-20}$ m.

Idea: can we exploit the link between quantum geometry and quantum information?

Motivation

Why use photons?

QFT in flat space

Can neither confirm nor predict what is realised in nature at the Planck scale by studying photon correlations.

Think: *analogue gravity models*

- new ways of looking at a problem
- opportunity for practical experiments within the analogue system

new insights

Motivation

Why use photons?

Can neither confirm nor predict

LETTER

https://doi.org/10.1038/s41586-019-1241-0

Observation of thermal Hawking radiation and its temperature in an analogue black hole

Juan Ramón Muñoz de Nova[1], Katrine Golubkov[1], Victor I. Kolobov[1] & Jeff Steinhauer[1]*

The entropy of a black hole[1] and Hawking radiation[2] should have the same temperature given by the surface gravity, within a numerical factor of the order of unity. In addition, Hawking radiation should have a thermal spectrum, which creates an information paradox[3,4]. However, the thermality should be limited by greybody factors[5], at the very least[6]. It has been proposed that the physics of Hawking radiation could be verified in an analogue system[7], an idea that has been carefully studied and developed theoretically[8–18]. Classical a real black hole. In a Bose–Einstein condensate, the dispersion relation is linear in the low-energy limit. Thus, we should create an analogue black hole with sufficiently low Hawking temperature that the radiation is in the linear regime of the dispersion relation. We can then test whether the emitted Hawking radiation obeys the prediction of equation (1). There are several theoretical works suggesting that this should be the case. Using previous analytical results for a system similar to this experiment[13], we find that equation (1) gives an accurate prediction for

❖ JRM de Nova, K Golubkov, VI Kolobov, J Steinhauser, *Nature* **569**, 688-691 (2019).

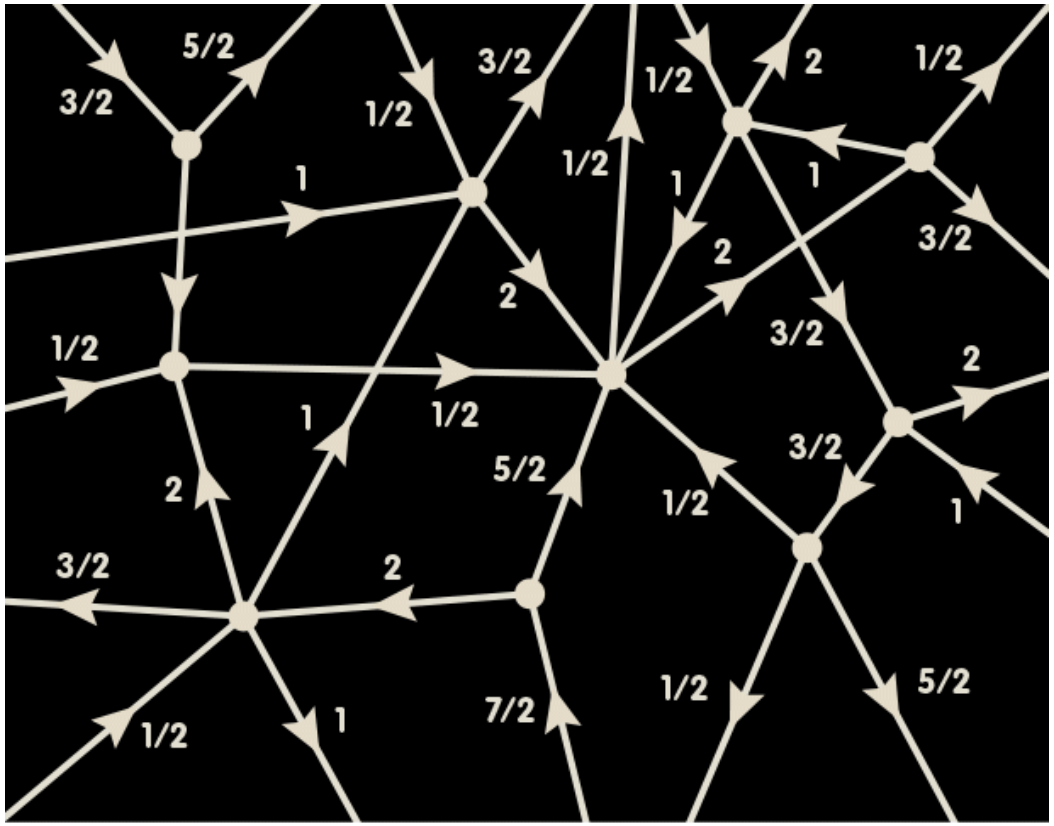
Think: *analogue gravity models*

- new ways of looking at a problem
- opportunity for practical experiments within the analogue system

➡ **new insights**

Analogue models for LQG?

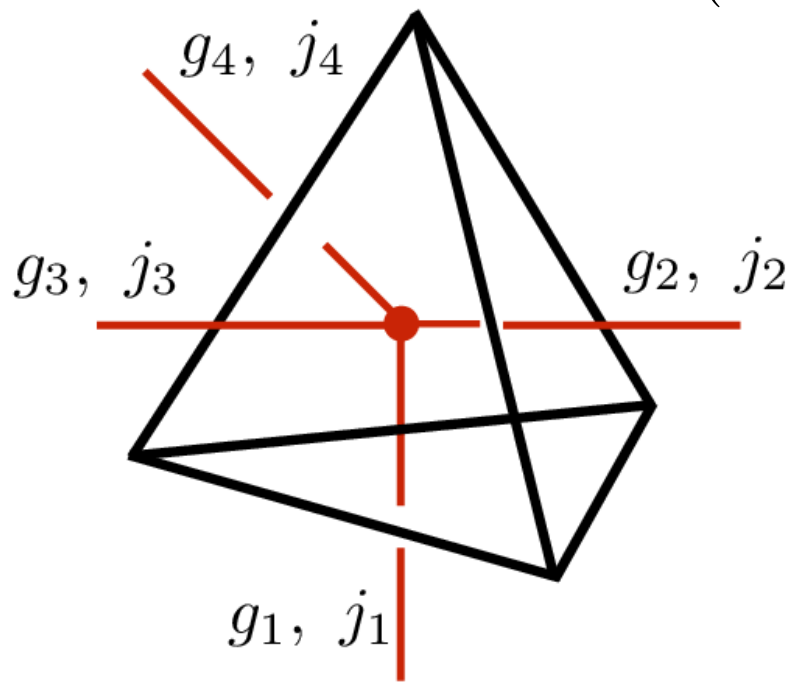
The fabric of space: spin networks



Nodes + Lines + Arrows + Labels = Spin network

Dual space:

$$\hat{V} = \left(\hat{q}^{\dagger}\hat{q}\right)^{1/4}$$



nodes ⟷ **volumes**

links ⟷ **areas**

❖ T Thiemann, *The fabric of space: spin networks*, Einstein Online **Band 01** (2005), 01-1029.

Motivation

1. Can we simulate discrete tetrahedra volumes, including fluctuations?

How many photons?

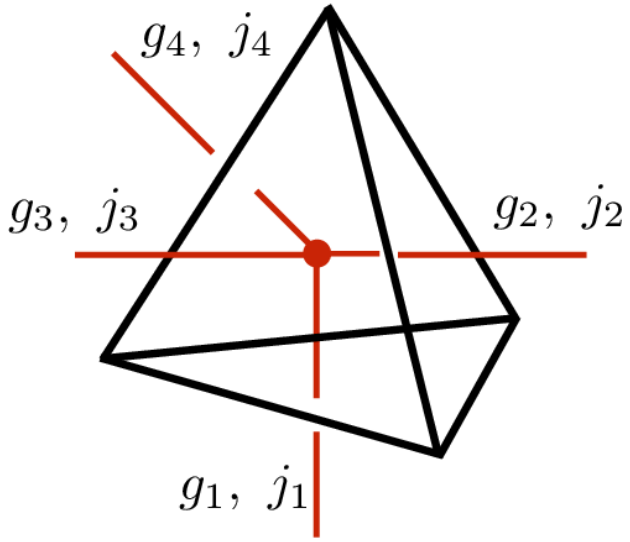
Procedure?

Experimental setup?

2. Entropic quantities?

Entanglement entropy?

etc.



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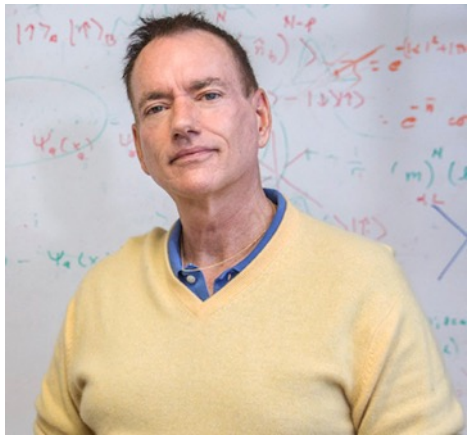

Jonathan P. Downling

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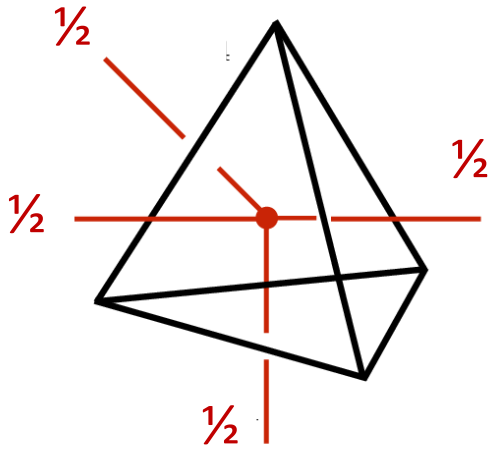
Collaborators

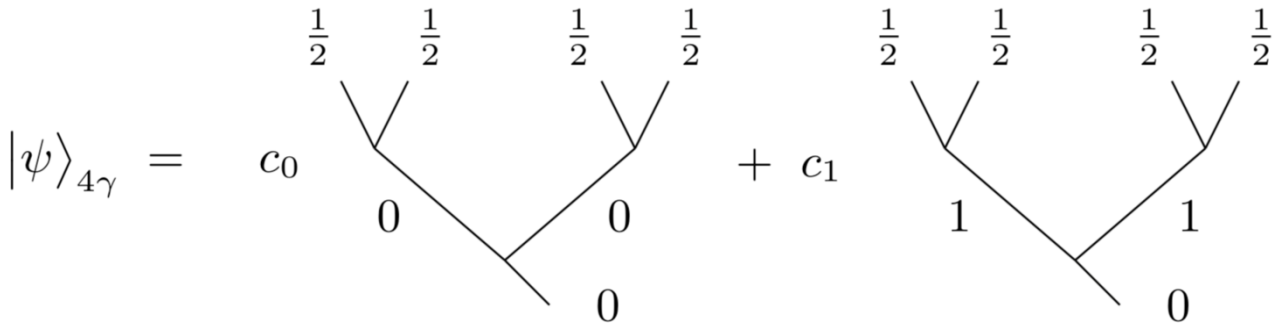
Four photon state spin network

– a simple example with fixed volume



$$|\psi\rangle_{4\gamma} = c_0 |\psi\rangle_{\substack{4\gamma \\ k=0}} + c_1 |\psi\rangle_{\substack{4\gamma \\ k=1}}$$

Diagrammatically:



Written in terms of spin-½ projections:

$$|\psi\rangle_{\substack{4\gamma \\ k=0}} = \frac{1}{2}\Big(|\downarrow\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle\Big)$$

$$|\psi\rangle_{\substack{4\gamma \\ k=1}} = \frac{1}{\sqrt{3}}\Big(|\downarrow\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\downarrow\rangle\Big) - \frac{1}{2\sqrt{3}}\Big(|\downarrow\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle\Big)$$

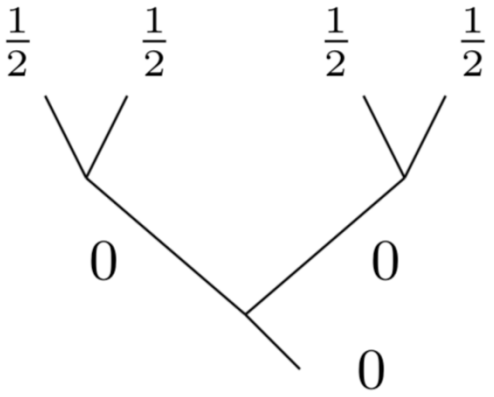
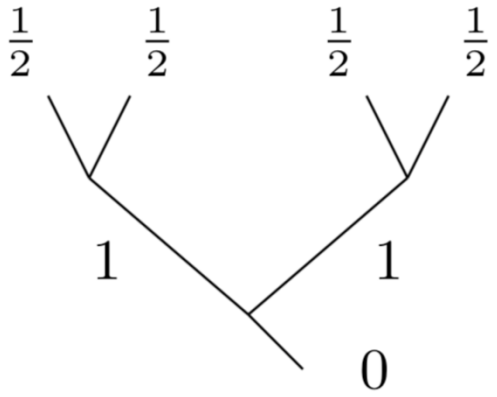
Four photon state spin network

– a simple example with fixed volume

$$|\psi\rangle_{4\gamma} = c_0 |\psi\rangle_{\substack{4\gamma \\ k=0}} + c_1 |\psi\rangle_{\substack{4\gamma \\ k=1}}$$

$$\begin{aligned}\mathcal{H}_{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}} &= \mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \\ &= (0 \oplus 1) \otimes (0 \oplus 1) = 0 \oplus 1 \oplus 1 \oplus 0 \oplus 1 \oplus 2\end{aligned}$$

$$\mathcal{K}_{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}} = \mathrm{Inv}_{\mathrm{SU}(2)}\left[\mathcal{H}_{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}}\right] \Rightarrow \dim \mathcal{K}_{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}} = 2$$

$|\psi\rangle_{4\gamma} =$ c_0  $+ c_1$ 

Matrix rep. of $\hat{V}$ is 2 dim.

2 EVs

Four photon state spin network

– a simple example with fixed volume

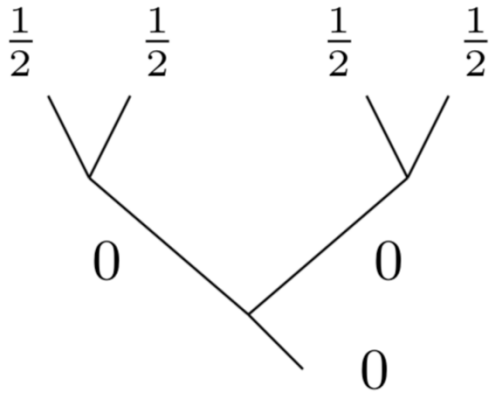
$$\hat{V} = \begin{bmatrix} \frac{\sqrt[4]{3}}{2} & 0 \\ 0 & \frac{\sqrt[4]{3}}{2} \end{bmatrix}$$

$$|\psi\rangle_{4\gamma} = c_0 |\psi\rangle^{4\gamma}_{k=0} + c_1 |\psi\rangle^{4\gamma}_{k=1}$$

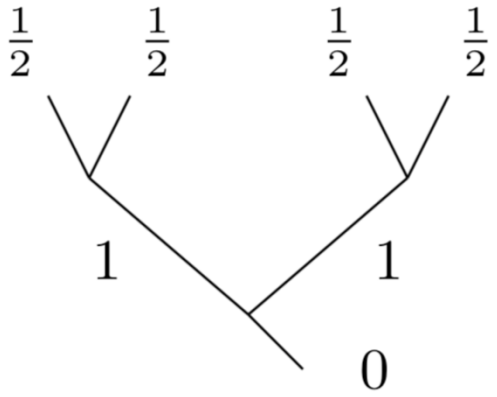
$$\lambda_1 = \lambda_2 = \langle\psi|\hat{V}|\psi\rangle = \frac{\sqrt[4]{3}}{2}$$

Spectrum of $\hat{V}$ is twofold degenerate.

$$|\psi\rangle_{4\gamma} = c_0$$



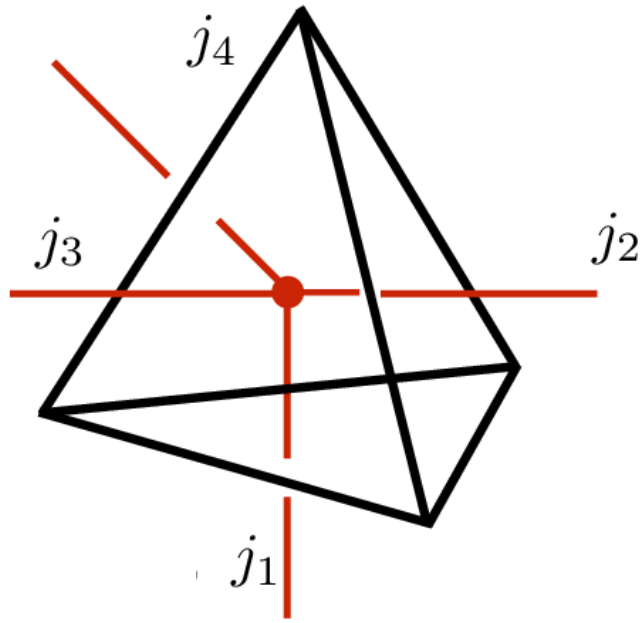
$$+ c_1$$



Cannot distinguish geometric configurations.

Calculating the volume operator

Dual space:



$$\hat{V} = \left(\hat{q}^\dagger \hat{q}\right)^{1/4}$$

$$\hat{q} = \begin{bmatrix} 0 & -a_1 & 0 & \dots & 0 \\ a_1 & 0 & -a_2 & & \vdots \\ 0 & a_2 & 0 & & \\ \vdots & & & \ddots & -a_{n-1} \\ 0 & \dots & & a_{n-1} & 0 \end{bmatrix}$$

$$a_k = \left\langle j_{12}^{\min} + k \middle| \hat{q} \middle| j_{12}^{\min} + k - 1 \right\rangle = \frac{-4}{\sqrt{4k^2-1}} \Delta\left(j_1 + \frac{1}{2}, j_2 + \frac{1}{2}, k\right) \Delta\left(j_3 + \frac{1}{2}, j_4 + \frac{1}{2}, k\right)$$

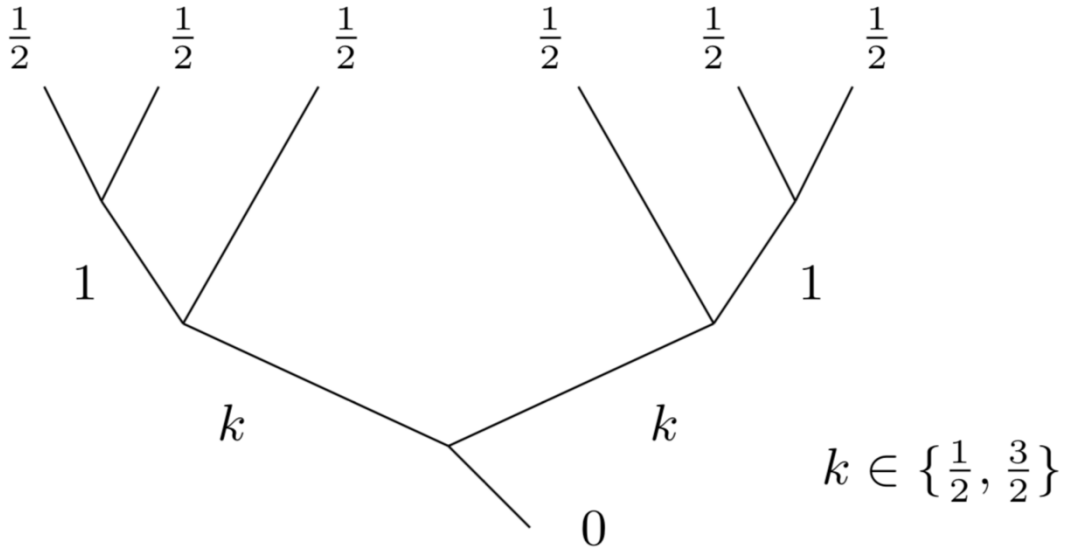
$$j_{12}^{\min} = \max\left[\left|j_1 - j_2\right|, \left|j_3 - j_4\right|\right]$$

$$j_{12}^{\max} = \min\left[\left|j_1 + j_2\right|, \left|j_3 + j_4\right|\right]$$

$$\Delta(a,b,c) = \frac{1}{4}\sqrt{(a+b+c)(a+b-c)(a-b+c)(-a+b+c)}$$

Six photon state spin network

$$|\psi\rangle_{6\gamma} = c_{\frac{1}{2}} |\psi\rangle_{\substack{6\gamma \\ k=\frac{1}{2}}} + c_{\frac{3}{2}} |\psi\rangle_{\substack{6\gamma \\ k=\frac{3}{2}}}$$



$$\mathcal{H}_{1\frac{1}{2}\frac{1}{2}1} = 0 \oplus 1 \oplus 1 \oplus 2 \oplus 1 \oplus 2 \oplus 0 \oplus 1 \oplus 2 \oplus 3$$

$$\mathcal{K}_{1\frac{1}{2}\frac{1}{2}1} = \mathrm{Inv}_{\mathrm{SU}(2)}\left[\mathcal{H}_{1\frac{1}{2}\frac{1}{2}1}\right] \;\Rightarrow\; \dim \mathcal{K}_{1\frac{1}{2}\frac{1}{2}1} = 2$$

$$\hat{V} = \begin{bmatrix} \frac{1}{\sqrt[4]{2}} & 0 \\ 0 & \frac{1}{\sqrt[4]{2}} \end{bmatrix}$$

Spectrum of $\hat{V}$ is twofold degenerate.

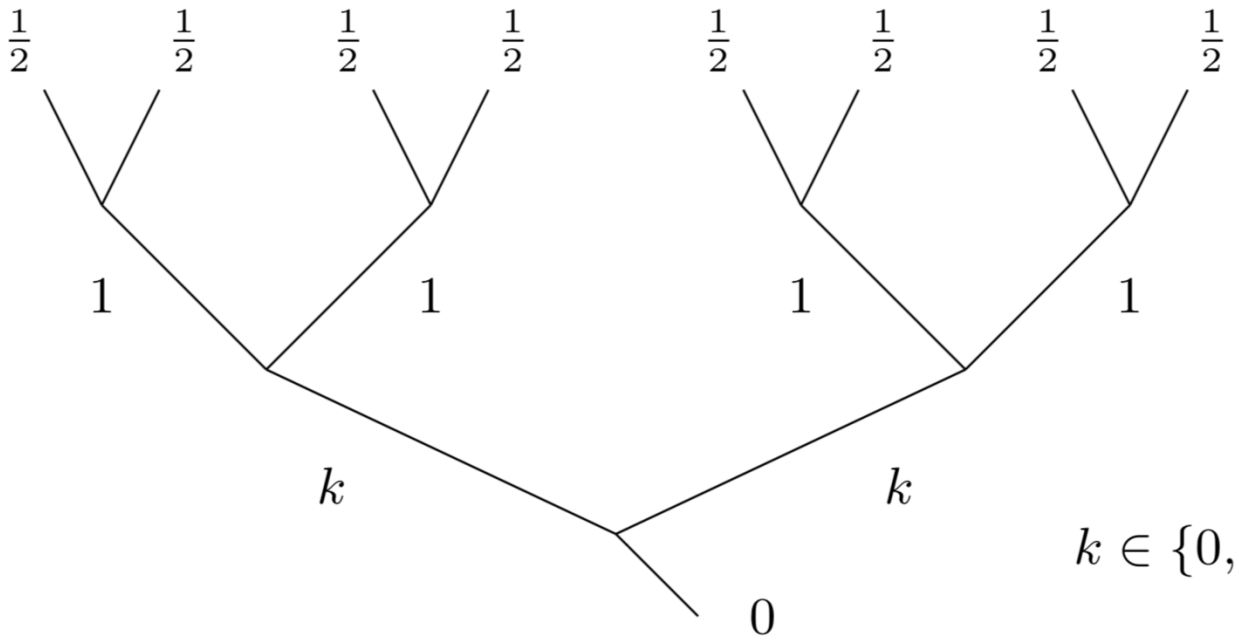
$$\lambda_1 = \lambda_2 = \langle\psi|\hat{V}|\psi\rangle = \frac{1}{\sqrt[4]{2}}$$

Cannot distinguish geometric configurations.

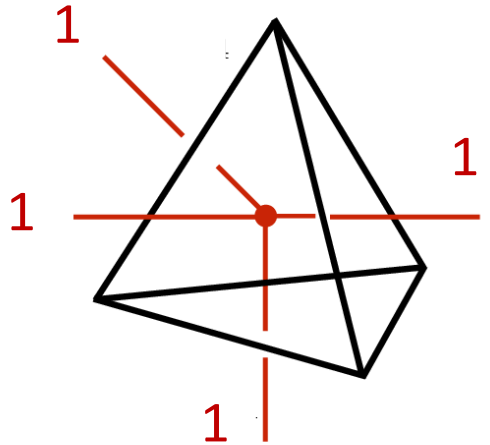
Eight photon state spin network

$$\mathcal{K}_{1111} = \mathrm{Inv}_{\mathrm{SU}(2)}\left[\mathcal{H}_{1111}\right] \;\Rightarrow\; \dim \mathcal{K} = 3$$

$$\left|\psi\right\rangle_{8\gamma} = c_0 \left|\psi\right\rangle_{\substack{8\gamma \\ k=0}} + c_1 \left|\psi\right\rangle_{\substack{8\gamma \\ k=1}} + c_2 \left|\psi\right\rangle_{\substack{8\gamma \\ k=2}}$$

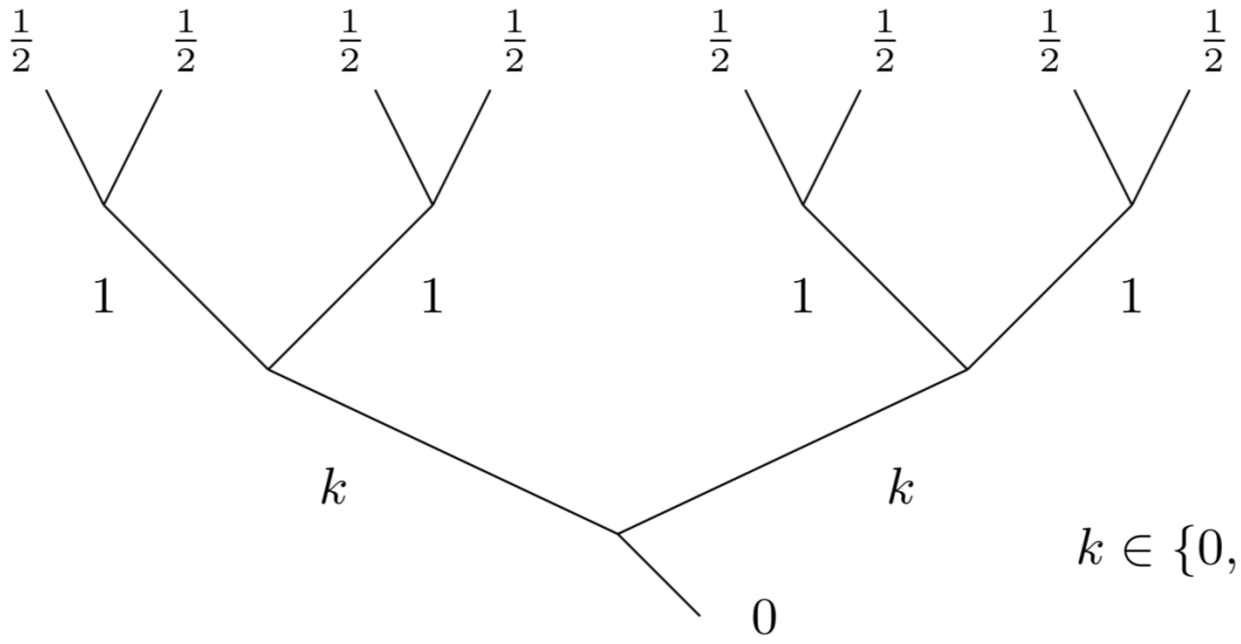


$k \in \{0, 1, 2\}$



Eight photon state spin network

$$|\psi\rangle_{8\gamma} = c_0 |\psi\rangle_{\substack{8\gamma \\ k=0}} + c_1 |\psi\rangle_{\substack{8\gamma \\ k=1}} + c_2 |\psi\rangle_{\substack{8\gamma \\ k=2}}$$



$k \in \{0, 1, 2\}$

$$\hat{V} = \begin{bmatrix} \frac{4}{3 \cdot 3^{3/4}} & 0 & -\frac{2\sqrt{5}}{3 \cdot 3^{3/4}} \\ 0 & 3^{1/4} & 0 \\ -\frac{2\sqrt{5}}{3 \cdot 3^{3/4}} & 0 & \frac{5}{3 \cdot 3^{3/4}} \end{bmatrix}$$

Eigenspectrum:

$$\lambda_1 = \lambda_2 = \sqrt[4]{3}$$

$$|\lambda_1\rangle = -\frac{2}{3}|0\rangle + \frac{\sqrt{5}}{3}|2\rangle \ ,$$

$$|\lambda_2\rangle = |1\rangle \ ;$$

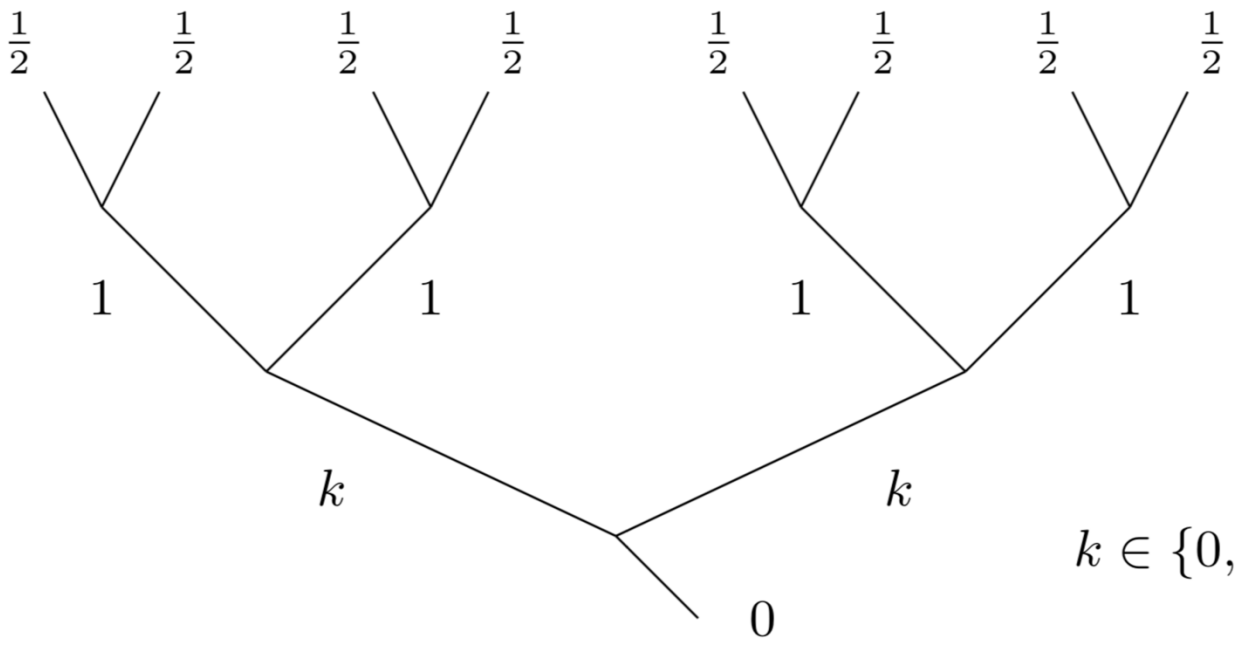
$$\lambda_3 = 0$$

$$|\lambda_3\rangle = \frac{\sqrt{5}}{3}|0\rangle + \frac{2}{3}|2\rangle \ .$$

Eight photon state spin network

$$|\psi\rangle_{8\gamma} = c_0 |\psi\rangle_{\substack{8\gamma \\ k=0}} + c_1 |\psi\rangle_{\substack{8\gamma \\ k=1}} + c_2 |\psi\rangle_{\substack{8\gamma \\ k=2}}$$

$$\hat{V} = \begin{bmatrix} \frac{4}{3\cdot 3^{3/4}} & 0 & -\frac{2\sqrt{5}}{3\cdot 3^{3/4}} \\ 0 & 3^{1/4} & 0 \\ -\frac{2\sqrt{5}}{3\cdot 3^{3/4}} & 0 & \frac{5}{3\cdot 3^{3/4}} \end{bmatrix}$$



$k \in \{0, 1, 2\}$

Obtain states with

$$\langle \Delta \hat{V} \rangle = \sqrt{\langle V^2 \rangle - \langle V \rangle^2} \neq 0$$

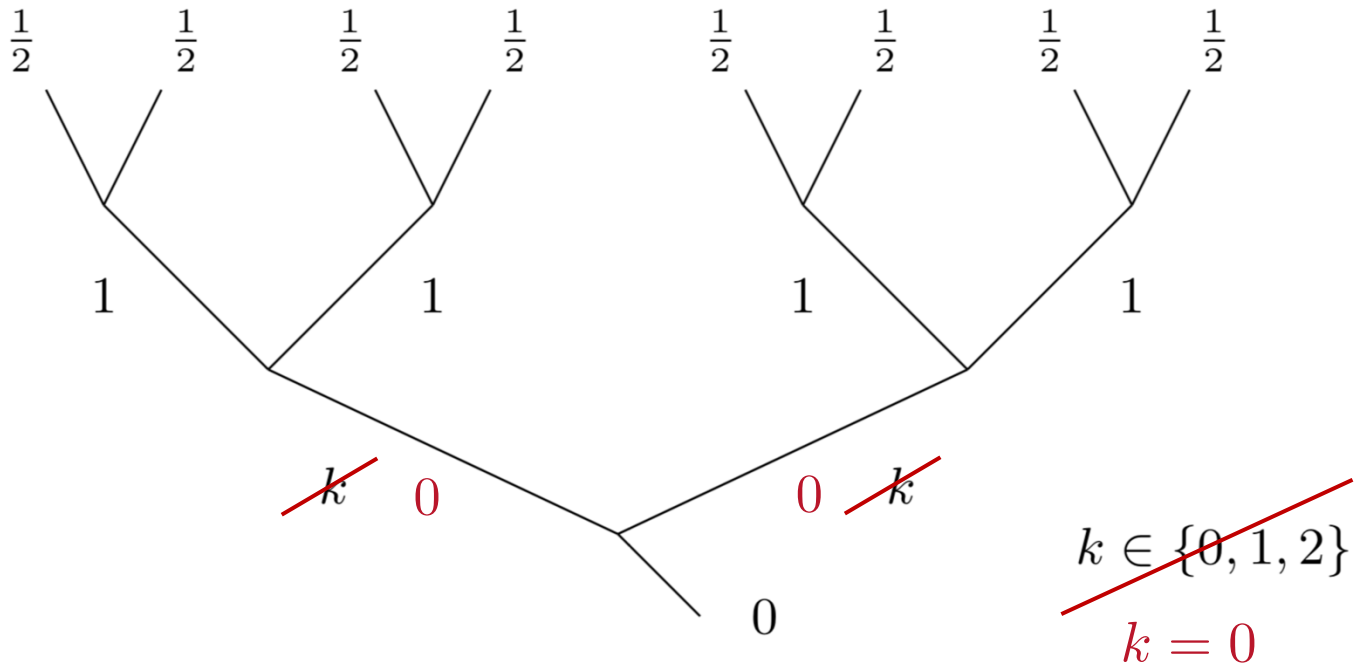
Corresponds to

fluctuating volume

in dual space.

Eight photon state spin network

$$|\psi\rangle_{\substack{8\gamma \\ k=0}} = \left[|\psi\rangle_{\substack{4\gamma \\ k=1}}\right]^{\otimes 2} = \left[\frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\downarrow\rangle\right) - \frac{1}{2\sqrt{3}}\left(|\downarrow\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle\right)\right]^{\otimes 2}$$



~~$k \in \{0, 1, 2\}$~~

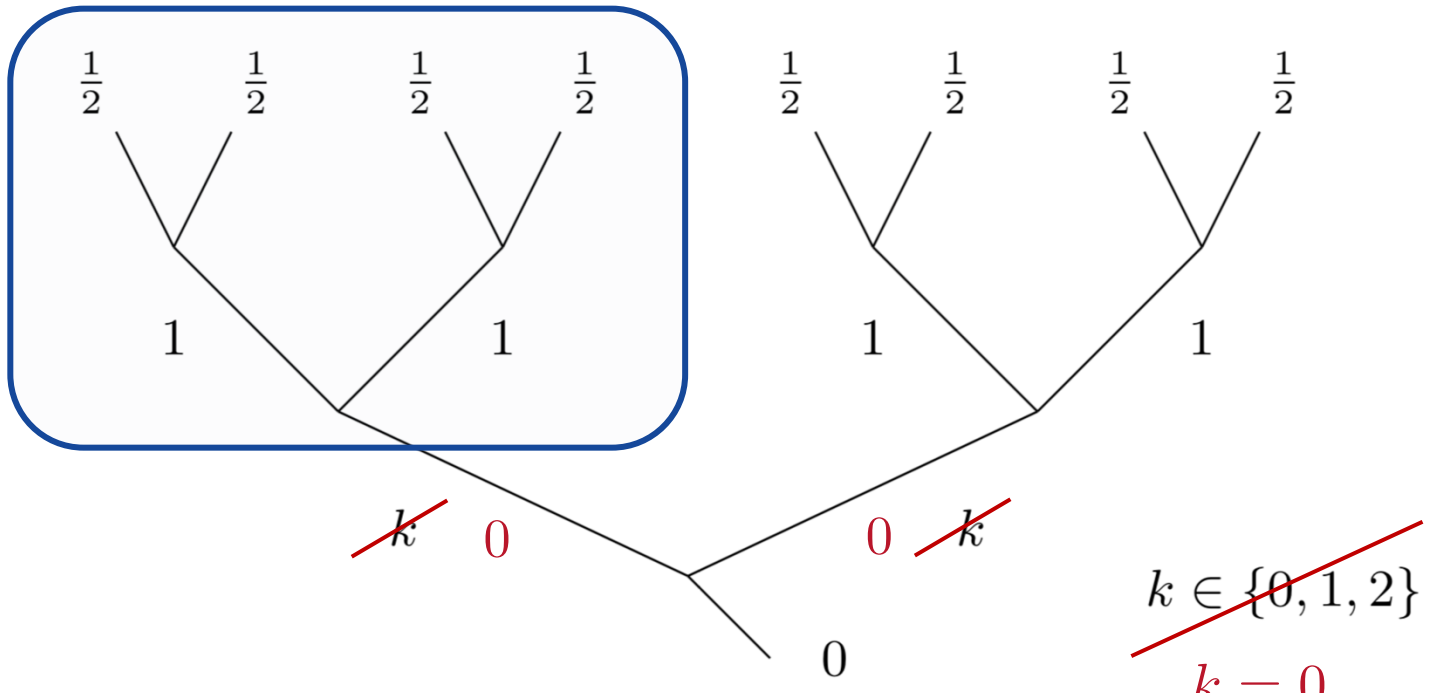
~~$k = 0$~~

For $k = 0$:

Can simulate eight photon state spin network using two identical copies of a four photon spin network.

Eight photon state spin network

$$|\psi\rangle_{\substack{8\gamma \\ k=0}} = \left[|\psi\rangle_{\substack{4\gamma \\ k=1}}\right]^{\otimes 2} = \left[\frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\downarrow\rangle\right) - \frac{1}{2\sqrt{3}}\left(|\downarrow\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle\right)\right]^{\otimes 2}$$



For $k = 0$:

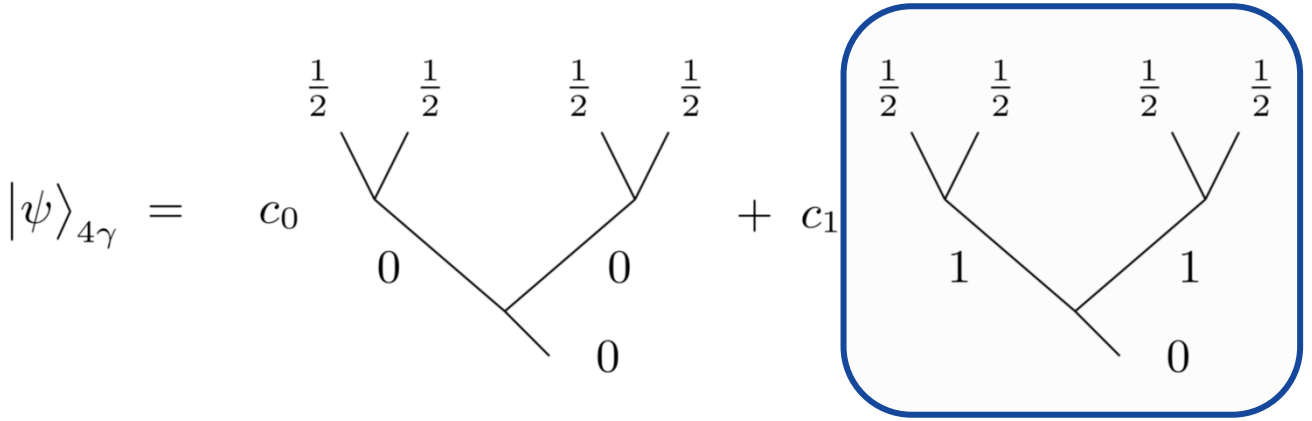
Can simulate eight photon state spin network using two identical copies of a four photon spin network.

Four photon state spin network

– a simple example with fixed volume

$$|\psi\rangle_{4\gamma} = c_0 |\psi\rangle_{\substack{4\gamma \\ k=0}} + c_1 |\psi\rangle_{\substack{4\gamma \\ k=1}}$$

Diagrammatically:



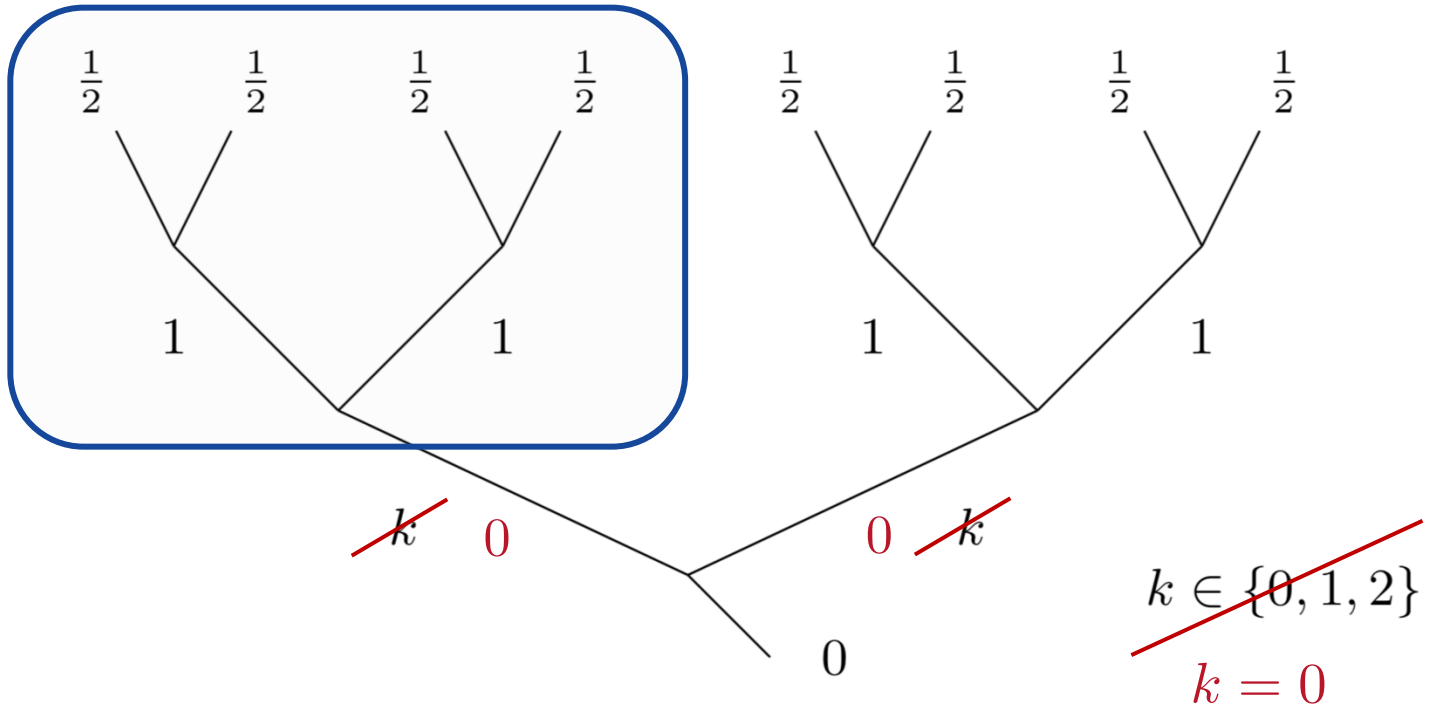
Written in terms of spin-½ projections:

$$|\psi\rangle_{\substack{4\gamma \\ k=0}} = \frac{1}{2}\Big(|\downarrow\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle\Big)$$

$$|\psi\rangle_{\substack{4\gamma \\ k=1}} = \frac{1}{\sqrt{3}}\Big(|\downarrow\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\downarrow\rangle\Big) - \frac{1}{2\sqrt{3}}\Big(|\downarrow\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle\Big)$$

Eight photon state spin network

$$|\psi\rangle_{\substack{8\gamma \\ k=0}} = \left[|\psi\rangle_{\substack{4\gamma \\ k=1}}\right]^{\otimes 2} = \left[\frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\downarrow\rangle\right) - \frac{1}{2\sqrt{3}}\left(|\downarrow\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle\right)\right]^{\otimes 2}$$



For $k = 0$:

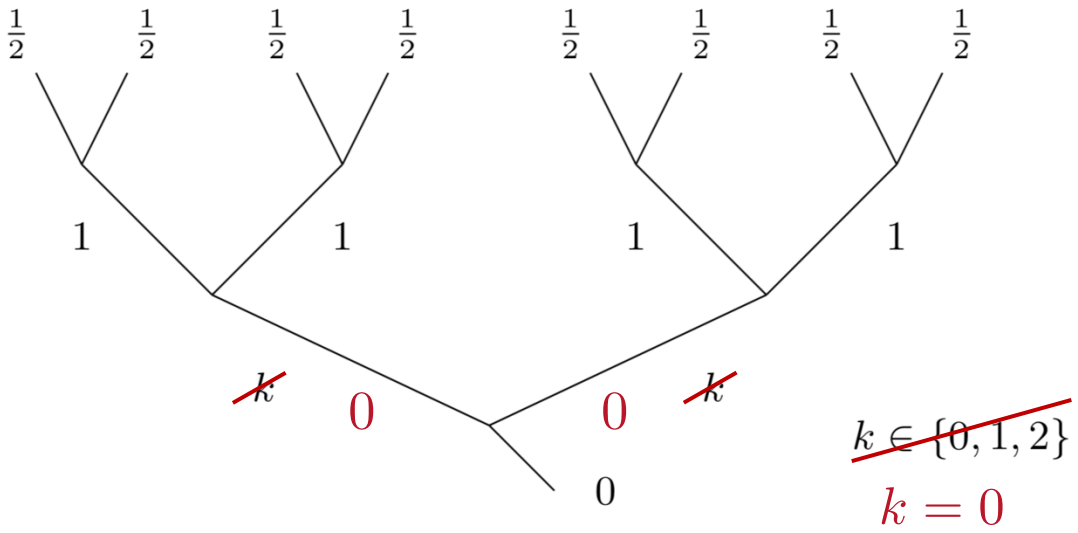
Can simulate eight photon state spin network using two identical copies of a four photon spin network.

But:

No entanglement between left and right branch.

Eight photon state spin network

Can we create entanglement?



Procedure:

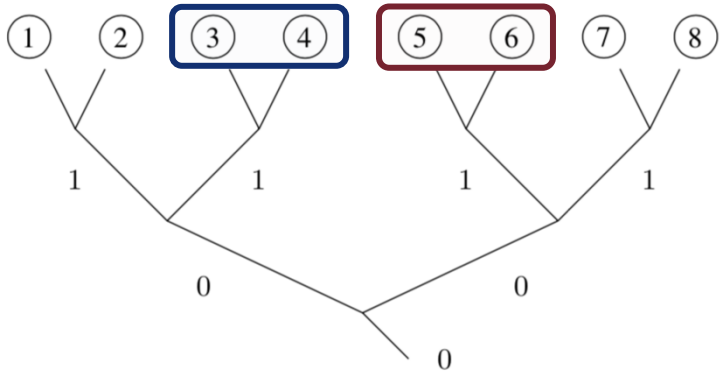
1. Apply SWAP operation to exchange photons from left branch with photons from right branch.
2. Project two photons (one from each branch) into a singlet state.
3. Perform Bell measurement to determine whether or not Bell/CHSH inequalities are violated.

Eight photon state spin network

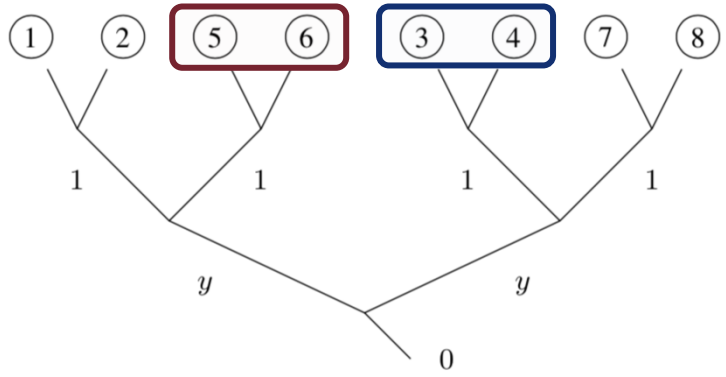
Can we create entanglement?

Procedure:

1. Apply SWAP operation to exchange photons from left branch with photons from right branch.



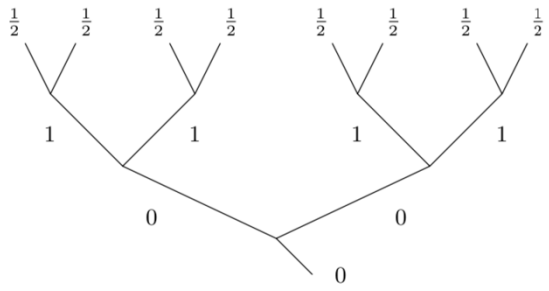
F move $\longrightarrow$ $\sum_y \sum_x \left(F_1^{111}\right)_y^x \left(F_1^{111}\right)_x^0$



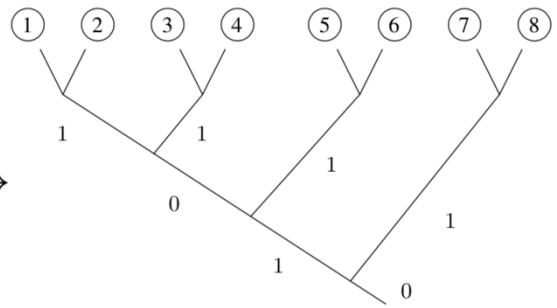
Calculation of *F* move coefficients: $\left(F_{abc}^d\right)_f^x = \sqrt{(2x+1)(2f+1)} \begin{Bmatrix} a & b & f \\ c & d & x \end{Bmatrix}$

Procedure:

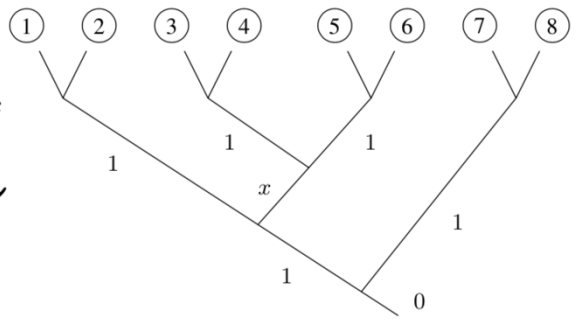
1. Apply SWAP operation to exchange photons from left branch with photons from right branch.



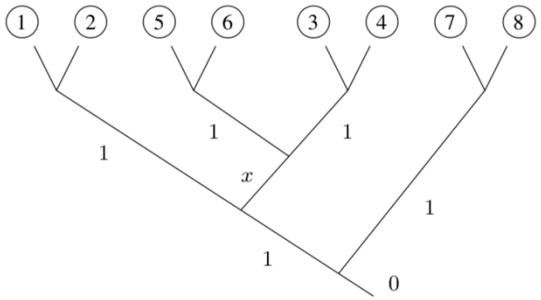
$\rightarrow$



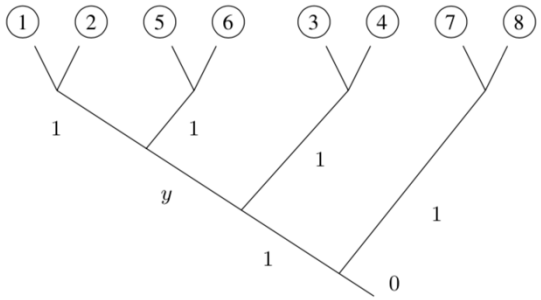
F move $\longrightarrow$ $\sum_x \underbrace{\left[\left(F_1^{111}\right)^{-1}\right]_0^x}_{=\left(F_1^{111}\right)_x^0}$



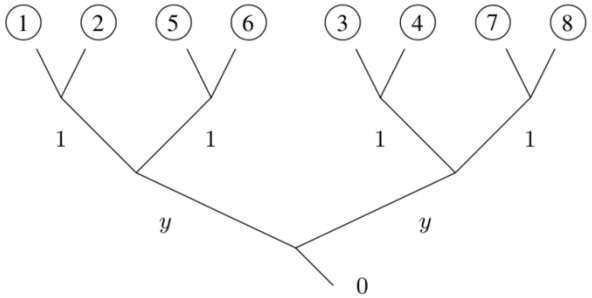
R move $\longrightarrow$ $(-1)\sum_x \underbrace{\left[\left(F_1^{111}\right)^{-1}\right]_0^x}_{=\left(F_1^{111}\right)_x^0}$



F move $\longrightarrow$ $\sum_y \sum_x \left(F_1^{111}\right)_y^x \left(F_1^{111}\right)_x^0$



F move $\longrightarrow$ $\sum_y \sum_x \left(F_1^{111}\right)_y^x \left(F_1^{111}\right)_x^0$

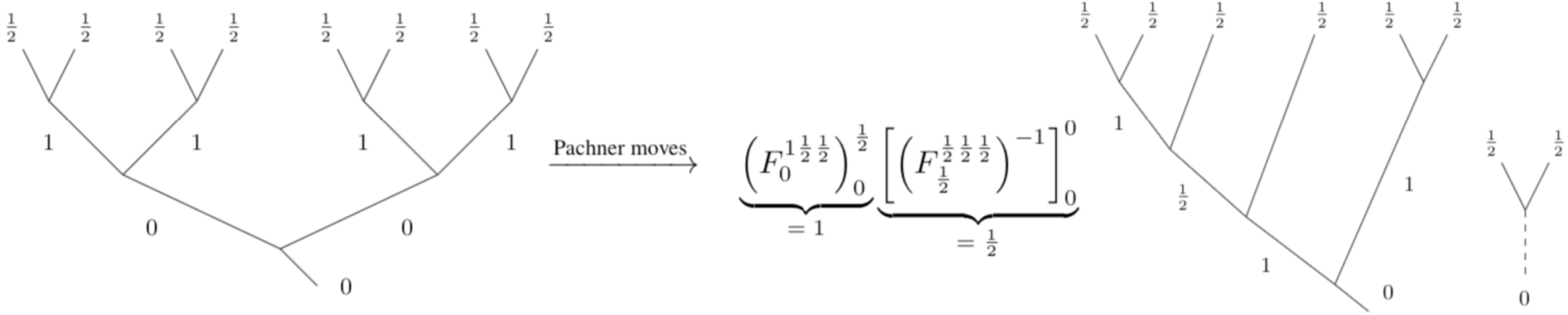


Eight photon state spin network

Can we create entanglement?

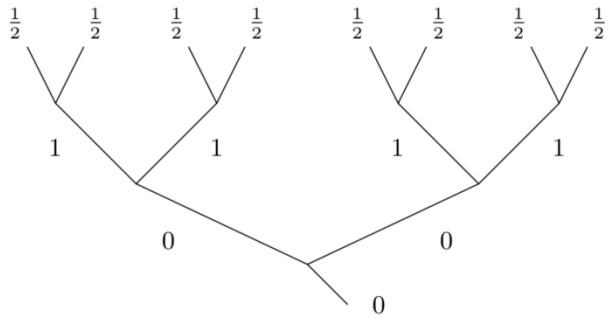
Procedure:

2. Project two photons (one from each branch) into a singlet state.

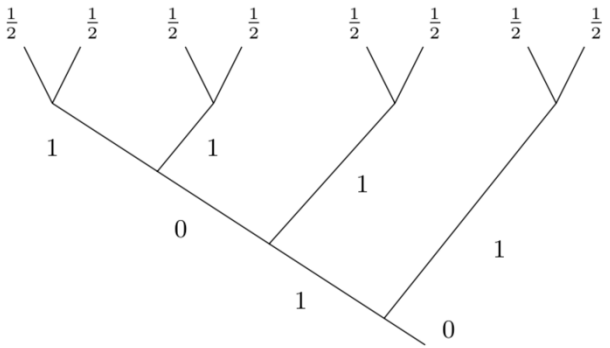


Procedure:

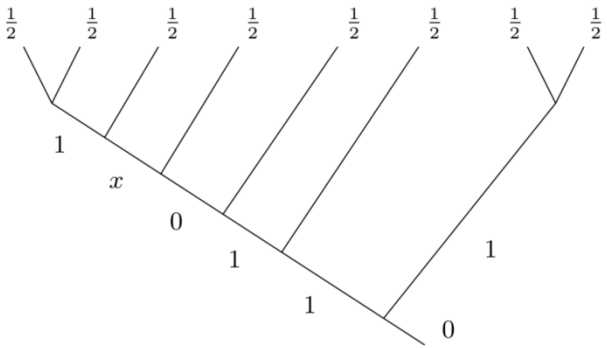
2. Project two photons (one from each branch) into a singlet state.



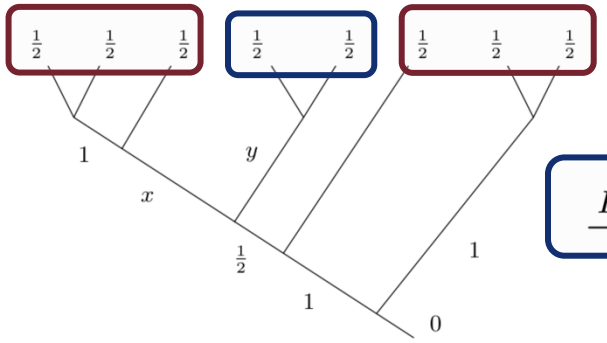
$\rightarrow$



$$\rightarrow \sum_{x} \left(F_0^{1\,\frac{1}{2}\,\frac{1}{2}}\right)_1^x$$

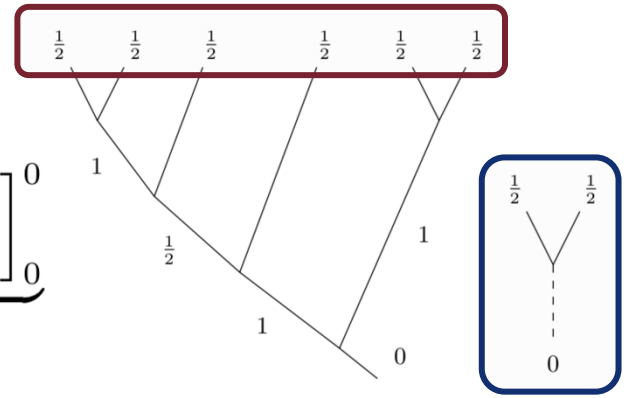


$$\rightarrow \sum_{x,y} \left(F_0^{1\,\frac{1}{2}\,\frac{1}{2}}\right)_1^x \left[\left(F_{\frac{1}{2}}^{x\,\frac{1}{2}\,\frac{1}{2}}\right)^{-1}\right]_0^y$$



$\xrightarrow{\hat{P}_{45}}$

$$\underbrace{\left(F_0^{1\,\frac{1}{2}\,\frac{1}{2}}\right)_1^{\frac{1}{2}}}_{=1} \underbrace{\left[\left(F_{\frac{1}{2}}^{\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}}\right)^{-1}\right]_0^0}_{=\frac{1}{2}}$$

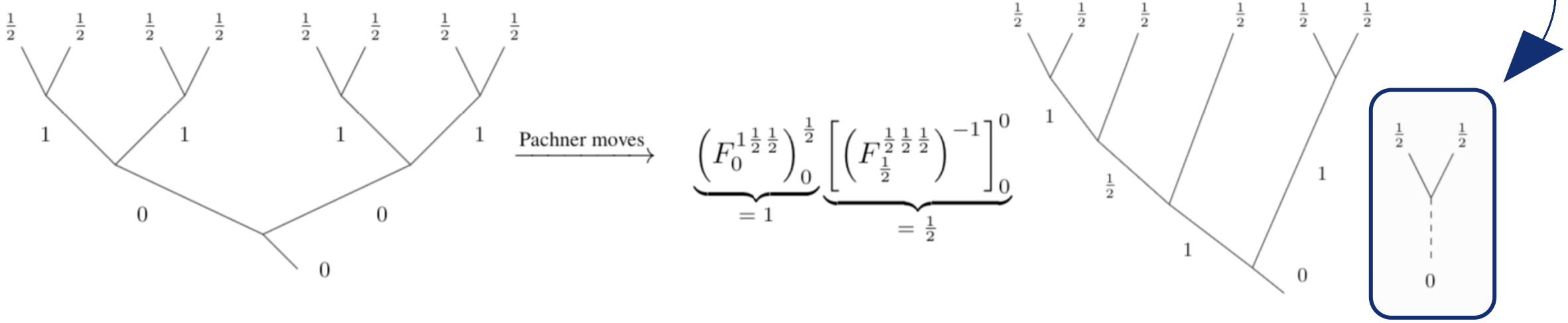


Eight photon state spin network

Can we create entanglement?

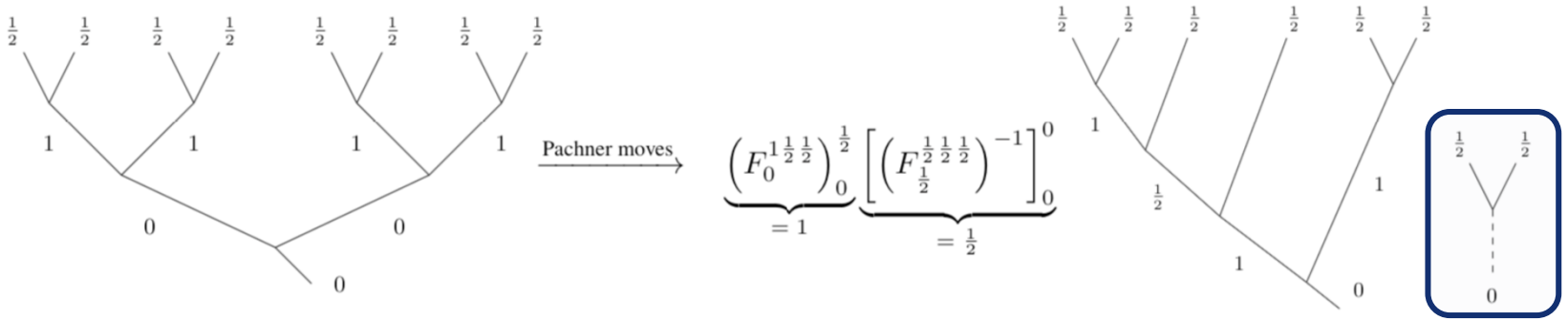
Procedure:

3. Perform Bell measurement to determine whether or not Bell/CHSH inequalities are violated.

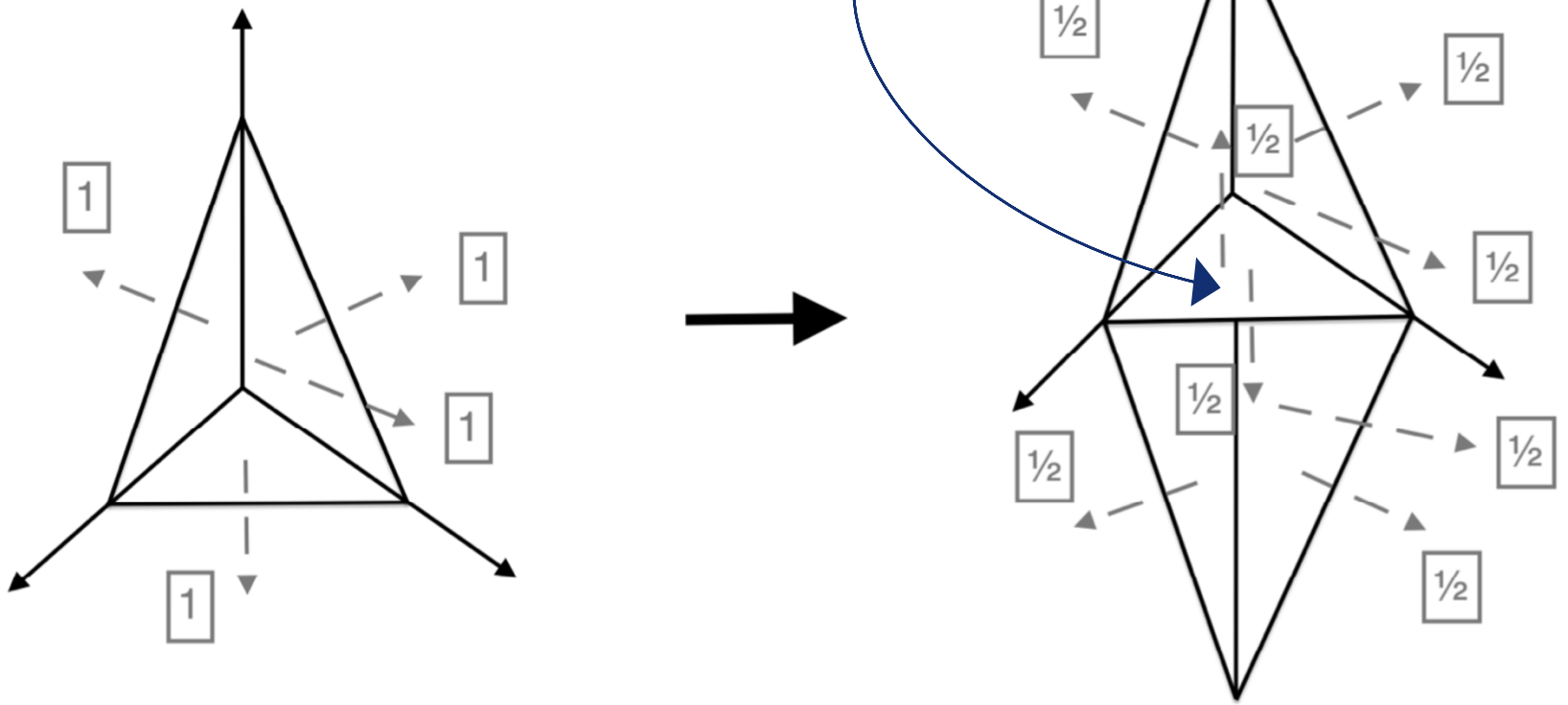


Procedure:

2. Project two photons (one from each branch) into a singlet state.



Geometrically:



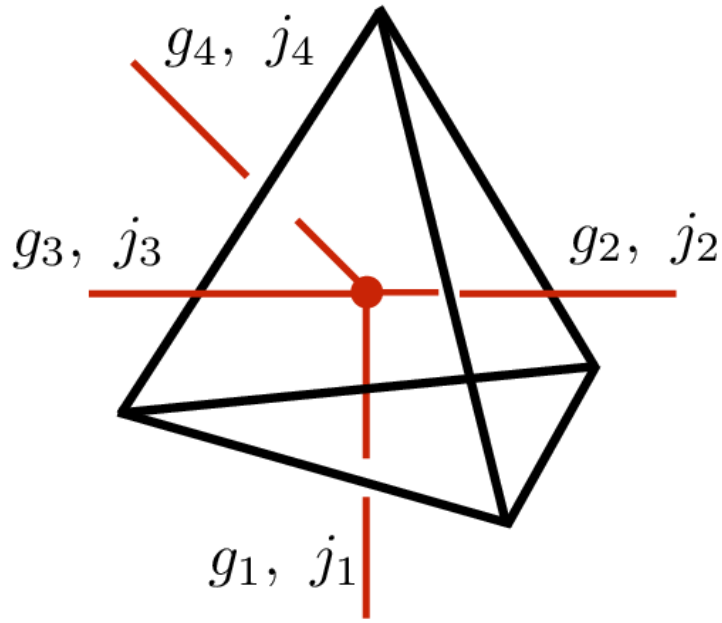
Backup slides

Calculating the volume operator

$$\hat{V} = \left(\hat{q}^\dagger \hat{q}\right)^{1/4}$$

$$\hat{q} = \begin{bmatrix} 0 & -a_1 & 0 & \dots & 0 \\ a_1 & 0 & -a_2 & & \vdots \\ 0 & a_2 & 0 & & \\ \vdots & & & \ddots & -a_{n-1} \\ 0 & \dots & & a_{n-1} & 0 \end{bmatrix}$$

Dual space:



$$a_k = \left\langle j_{12}^{\min} + k \middle| \hat{q} \middle| j_{12}^{\min} + k - 1 \right\rangle = \frac{-4}{\sqrt{4k^2-1}} \Delta\left(j_1 + \frac{1}{2}, j_2 + \frac{1}{2}, k\right) \Delta\left(j_3 + \frac{1}{2}, j_4 + \frac{1}{2}, k\right)$$

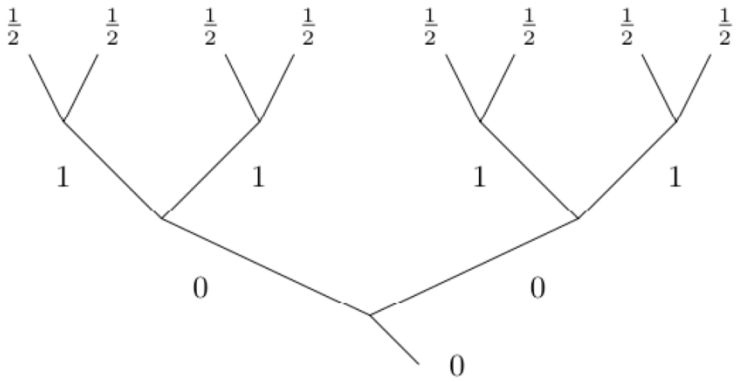
$$j_{12}^{\min} = \max\left[\left|j_1 - j_2\right|, \left|j_3 - j_4\right|\right]$$

$$j_{12}^{\max} = \min\left[\left|j_1 + j_2\right|, \left|j_3 + j_4\right|\right]$$

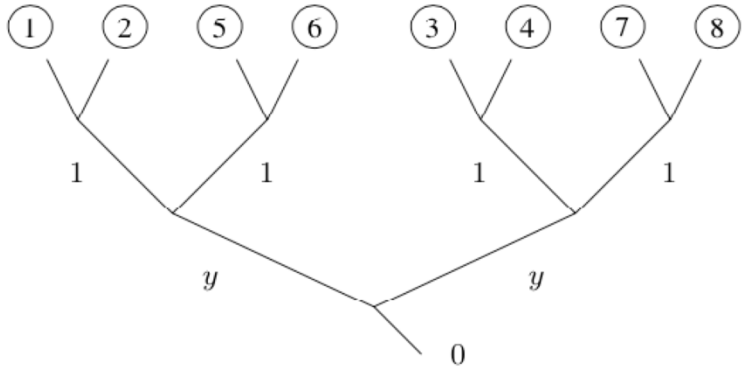
$$\Delta(a,b,c) = \frac{1}{4}\sqrt{(a+b+c)(a+b-c)(a-b+c)(-a+b+c)}$$

Procedure:

1. Apply SWAP operation to exchange photons from left branch with photons from right branch.



$$\xrightarrow{\text{F move}} \sum_y \sum_x \left(F_1^{111}\right)_y^x \left(F_1^{111}\right)_x^0$$

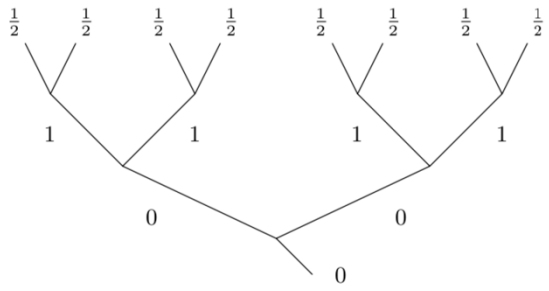


Calculation of F move coefficients:

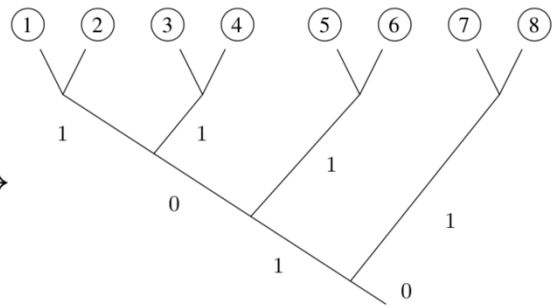
$$\left(F_{abc}^d\right)_f^x = \sqrt{(2x+1)(2f+1)} \begin{Bmatrix} a & b & f \\ c & d & x \end{Bmatrix}$$

Procedure:

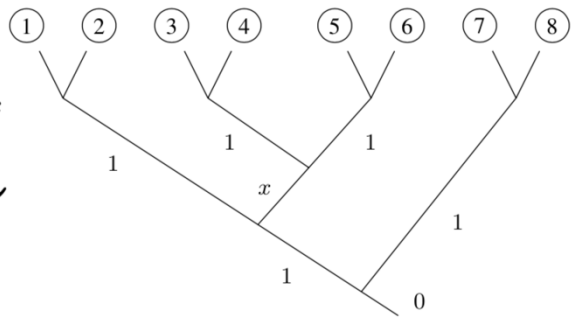
1. Apply SWAP operation to exchange photons from left branch with photons from right branch.



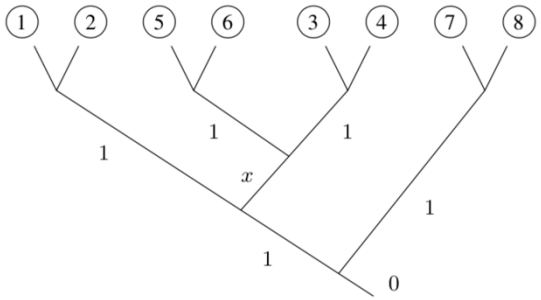
$\rightarrow$



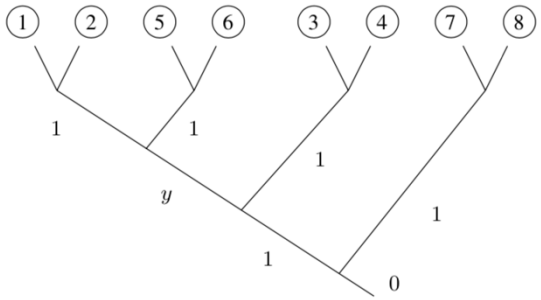
F move $\longrightarrow$ $\sum_x \underbrace{\left[\left(F_1^{111}\right)^{-1}\right]_0^x}_{=\left(F_1^{111}\right)_x^0}$



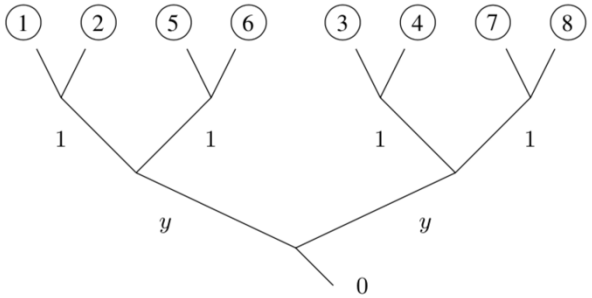
R move $\longrightarrow$ $(-1)\sum_x \underbrace{\left[\left(F_1^{111}\right)^{-1}\right]_0^x}_{=\left(F_1^{111}\right)_x^0}$



F move $\longrightarrow$ $\sum_y \sum_x \left(F_1^{111}\right)_y^x \left(F_1^{111}\right)_x^0$

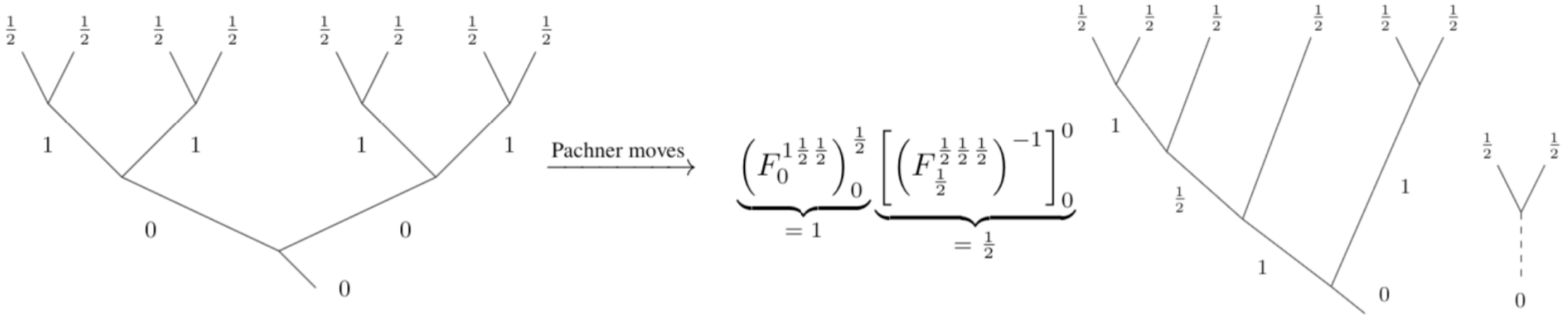


F move $\longrightarrow$ $\sum_y \sum_x \left(F_1^{111}\right)_y^x \left(F_1^{111}\right)_x^0$



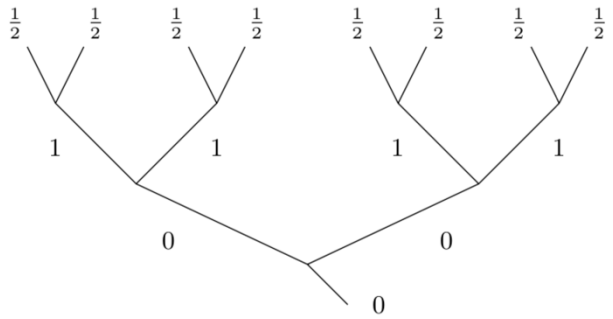
Procedure:

2. Project two photons (one from each branch) into a singlet state.

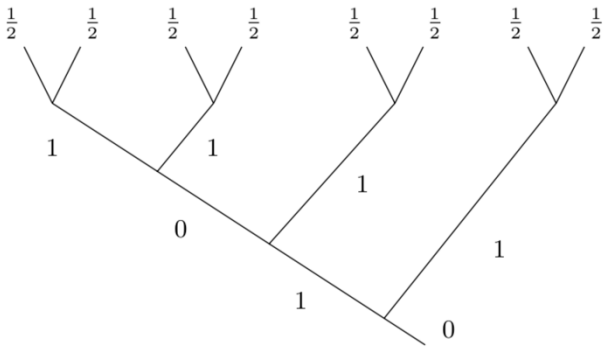


Procedure:

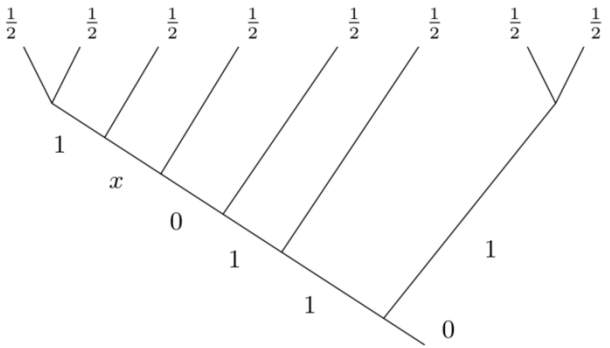
2. Project two photons (one from each branch) into a singlet state.



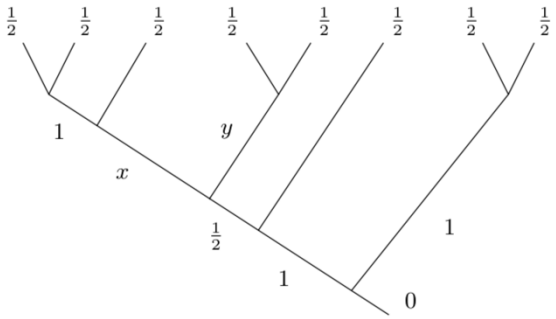
$\rightarrow$



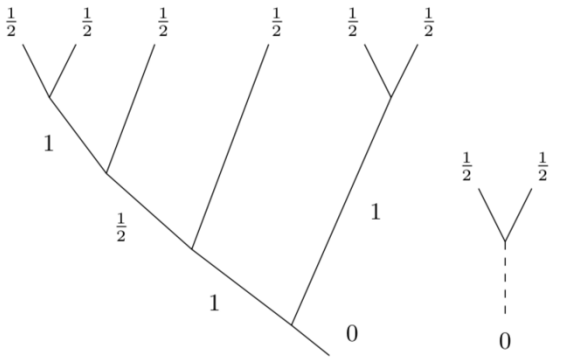
$$\rightarrow \sum_{x} \left(F_0^{1\,\frac{1}{2}\,\frac{1}{2}}\right)_1^{x}$$



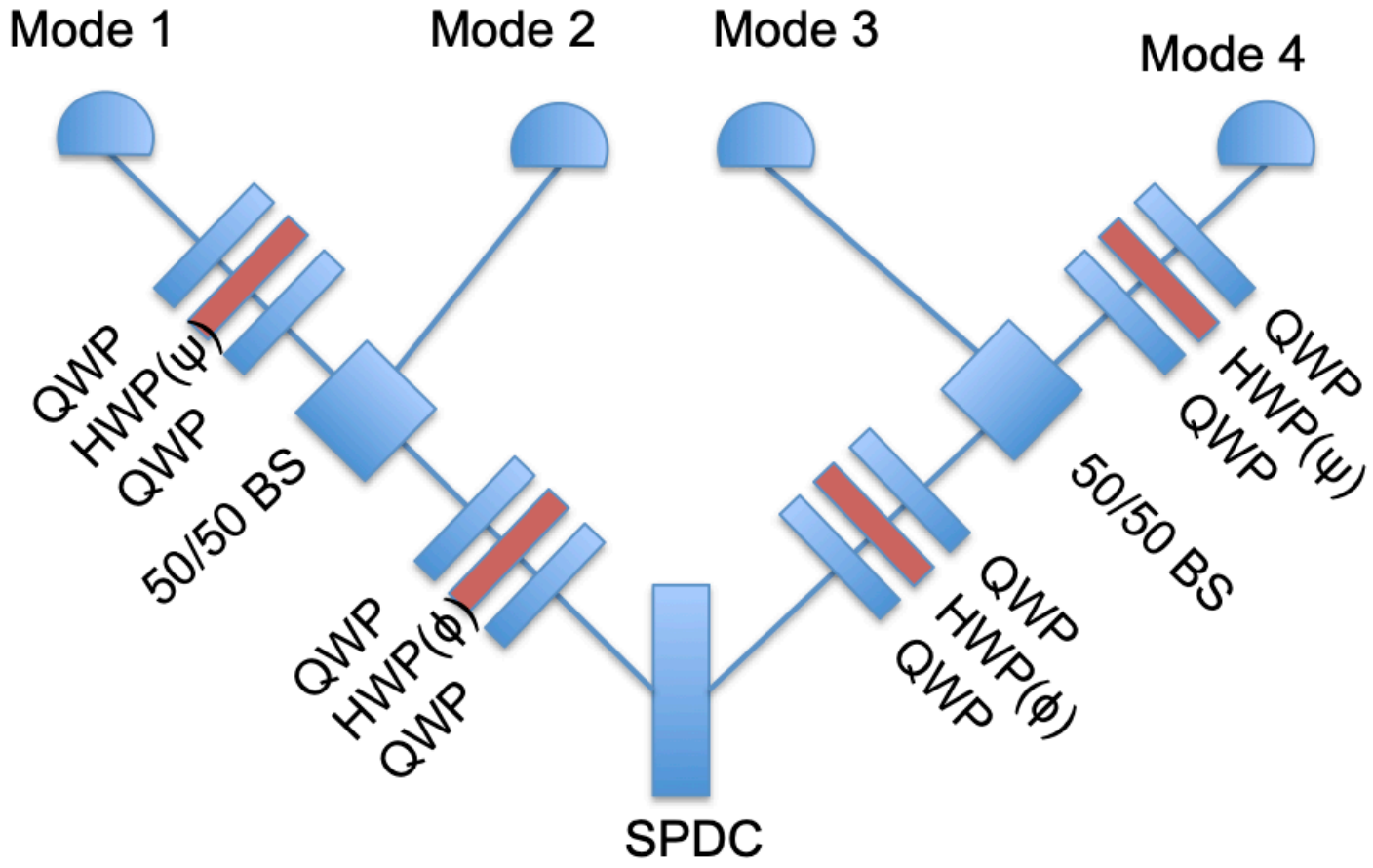
$$\rightarrow \sum_{x,y} \left(F_0^{1\,\frac{1}{2}\,\frac{1}{2}}\right)_1^{x} \left[\left(F_{\frac{1}{2}}^{x\,\frac{1}{2}\,\frac{1}{2}}\right)^{-1}\right]_0^{y}$$



$$\xrightarrow{\hat{P}_{45}} \underbrace{\left(F_0^{1\,\frac{1}{2}\,\frac{1}{2}}\right)_1^{\frac{1}{2}}}_{=1} \underbrace{\left[\left(F_{\frac{1}{2}}^{\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}}\right)^{-1}\right]_0^{0}}_{=\frac{1}{2}}$$



Eight photon state spin network – state creation



- Spatial modes at the output of the crystal are defined as modes 2 & 3.
- Post-selecting on one click at each of the detectors, the state is

$$-a^\dagger_{1h}a^\dagger_{2h}a^\dagger_{3v}a^\dagger_{4v} - a^\dagger_{1v}a^\dagger_{2v}a^\dagger_{3h} + \frac{1}{2}a^\dagger_{1h}a^\dagger_{2v}a^\dagger_{3h}a^\dagger_{4v}$$
$$+\frac{1}{2}a^\dagger_{1h}a^\dagger_{2v}a^\dagger_{3v}a^\dagger_{4h} + \frac{1}{2}a^\dagger_{1v}a^\dagger_{2h}a^\dagger_{3h}a^\dagger_{4v} + \frac{1}{2}a^\dagger_{1v}a^\dagger_{2h}a^\dagger_{3v}a^\dagger_{4h}$$

- Quarter-half-quarter wave plates on both sides of the beam splitter to produce the right phases.

Eight photon state produced via second order spontaneous parametric down-conversion.